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Maximally supersymmetric solutions of $D = 4$ $N = 2$ gauged supergravity

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Abstract: We determine and analyze maximally supersymmetric configurations in four-dimensional gauged $N = 2$ supergravity, preserving eight supercharges. These models include arbitrary electric gaugings in the vector- and hypermultiplet sectors. We present several examples of such solutions and connect some of them to vacuum solutions of flux compactifications in string theory.

Keywords: Supergravity Models, Flux compactifications, Superstring Vacua

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1 Introduction

It is of general interest to study four-dimensional supersymmetric string vacua and their low-energy effective supergravity descriptions. Firstly, in the context of flux compactifications and gauged supergravities, one is motivated by the problem of moduli stabilization and the properties of string vacua in which these moduli are stabilized. For some reviews on the topic, see [1–3]. Often, one focuses on supersymmetric vacua since there is better control over the dynamics of the theory, though for more realistic situations, e.g. in accelerating cosmologies, the vacuum must break all supersymmetry. Secondly, we are motivated to look for new versions of the AdS_4/CFT_3 correspondence. The recently proposed dualities studied in [4] are based on AdS_4 string vacua preserving 32 or 24 supersymmetries. Versions of the AdS_4/CFT_3 correspondence with less amount of supersymmetry are not yet well established (for some results on the correspondence in an $N = 2$ setting, see [5–7] and references therein), but are important for studying aspects of four-dimensional quantum gravity, and potentially also for certain condensed matter systems at criticality described by three-dimensional conformal field theories.

In this paper, we consider four-dimensional $N = 2$ gauged supergravities, and study the configurations that preserve maximal supersymmetry, i.e. eight supercharges. We only consider electric gaugings because magnetic gaugings require in addition massive tensor multiplets which have not been fully constructed yet. In the ungauged case, $N = 2$ models arise e.g. from Calabi-Yau compactifications of type II string theories, or $K3 \times T^2$ compactifications of the heterotic string. Both models are known to have a rich dynamical structure with controllable quantum effects in both vector- and hypermultiplet sectors that are relatively well understood. Gaugings in $N = 2$ supergravity are well studied and have a long history [8–13]. Their analysis in terms of string compactifications with fluxes started in [14], and is an ongoing research topic. For a (partial) list of references, see [15–21].

In the ungauged case, a complete classification of all the supersymmetric solutions already exists [22–24], while there have been also solutions in the gauged case for (abelian) vector multiplets [25]. We extend this by taking completely general vector- and hypermultiplet sectors. Since we concentrate only on the maximally supersymmetric solutions, we use different methods than the ones in the above references. In fact the space-time conditions we obtain for our solutions closely resemble other maximally supersymmetric solutions in different theories such as [26].

The plan of the paper is as follows. In section 2, we analyze the supersymmetry rules and derive the conditions for maximally supersymmetric vacua. The possible solutions divide in two classes of space-times, with zero scalar curvature and with negative scalar curvature, and we explicitly list all the possible outcomes. We give the lagrangian and the scalar potential for the obtained vacua in section 3, paying special attention to the Chern-Simons-like term determined by the c-tensor of the electric gauging. This term generically exists in $N = 2$ supergravity and string theory compactifications and we show how it influences the maximally supersymmetric vacua. In section 4, we discuss explicit cases from string theory compactifications and general supergravity considerations that exemplify the use of our maximal supersymmetry conditions. We have left the definition of our conventions and notations for the appendices, where we also present some intermediate and final formulae that are important for our results.

2 N=2 supersymmetry rules

We consider in this section vector multiplets, hypermultiples and the gravitational multiplet, with arbitrary electric gaugings, and will mostly follow the notation of [11]. For completeness, a list of conventions is given in appendix A.

As is well known, the vector multiplet sector is characterized by holomorphic sections $X^\Lambda(z)$ and $F_\Lambda(z), \Lambda = 0, 1, \ldots, n_V$, and the scalars $z^i; i = 1, \ldots, n_V$ parametrize a special Kähler manifold with Kähler potential

$$\mathcal{K}(z, \bar{z}) = -\ln\left[i(\bar{X}^\Lambda(\bar{z})F_\Lambda(z) - X^\Lambda(z)\bar{F}_\Lambda(\bar{z}))\right] . \tag{2.1}$$

When a prepotential exists, it is given by $2F = X^\Lambda F_\Lambda$. It should be homogeneous of second degree, and one must have that $F_\Lambda(X) = \partial F(X)/\partial X^\Lambda$. Our general analysis does not assume the existence of a prepotential.

The scalars in the hypermultiplet sector parametrize a quaternion-Kähler manifold, whose metric can be expressed in terms of quaternionic vielbeine. In local coordinates $q^u; u = 1, \ldots, 4n_H$, we have

$$h_{\rm uv}(q) = \mathcal{U}_u^{A\alpha}(q)\,\mathcal{U}_v^{B\beta}(q)\,\mathbb{C}_{\alpha\beta}\,\epsilon_{AB}\ , \tag{2.2}$$

where $\mathbb{C}_{\alpha\beta}, \alpha, \beta = 1, \ldots, 2n_H$ and $\epsilon_{AB}, A, B = 1, 2$ are the antisymmetric symplectic and SU(2) metrics, respectively. The value of the Ricci-scalar curvature of the quaternionic metric is always negative and fixed in terms of Newton's coupling constant κ. In units in which $\kappa^2 = 1$, which we will use in the remainder of this paper, we have

$$R(h) = -8n_H(n_H + 2)\ . \tag{2.3}$$

The analysis of maximally supersymmetric configurations does not rely on the form of the action, only on the supersymmetry variations and the equations of motion. Nevertheless, it is relevant to know what is the value of the scalar potential evaluated at such a configuration. We therefore turn to the properties of the Lagrangian in the next section.

It can be seen by inspection that the maximally supersymmetric configurations[1] are purely bosonic, and the fermions need to be zero. This follows from the supersymmetry variations of the bosonic fields, which can be read off from [11]. Therefore, we can restrict ourselves to the supersymmetry variations of the fermions only.

2.1 Gauginos

The number of vector multiplets is denoted by n_V, and in $N = 2$ special geometry, it is convenient to introduce indices $\Lambda = 0, 1, \ldots, n_V$ and $i = 1, \ldots, n_V$. The two fermions with positive chirality in a vector multiplet are denoted by λ^{iA}, with $A = 1, 2$. Complex conjugation changes the chirality and lowers the SU$(2)_R$ indices $A, B, \ldots$. See appendix A for more on our notations and conventions. Under gauged supersymmetry, with coupling constant g, the gauginos transform into

$$\delta_\varepsilon \lambda^{iA} = i\nabla_\mu z^i \gamma^\mu \varepsilon^A + G^{-i}_{\mu\nu}\gamma^{\mu\nu}\epsilon^{AB}\varepsilon_B + gW^{iAB}\varepsilon_B\ , \tag{2.4}$$

up to terms that are higher order in the fermions and which vanish for purely bosonic configurations. The supersymmetry parameters are denoted by ε^A. They have negative chirality and under complex conjugation $\varepsilon_A \equiv (\varepsilon^A)^*$, chirality is flipped since in our conventions γ_5 is hermitian but purely imaginary. We explain more on the quantities in (2.4) as we go along.

A maximally supersymmetric configuration preserves the full eight supercharges, hence the variation of the fermions should vanish for all choices of the supersymmetry parameters.

[1] In this paper we use interchangeably the terms maximally supersymmetric configurations and BPS configurations, meaning the field values that are invariant under all eight supercharges in the theory.

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Since at each point in spacetime they are linearly independent, the first term on the right hand side of (2.4) must vanish separately from the others,

$$\nabla_\mu z^i \equiv \partial_\mu z^i + g A_\mu^\Lambda k_\Lambda^i = 0 \ . \tag{2.5}$$

It implies the integrability condition[2]

$$F_{\mu\nu}^\Lambda \, k_\Lambda^i = 0 \,, \tag{2.6}$$

and complex conjugate. Here, $F_{\mu\nu}^\Lambda$ is the full non-abelian field strength.

The z^i are the complex scalars of the vector multiplets, and A_μ^Λ are the gauge fields (including the graviphoton). These scalars parametrize a special Kähler manifold which may have a group of isometries. To commute with supersymmetry, these isometries need to be holomorphic, and we denote the Killing vector fields by $k_\Lambda(z)$. Under the isometry, the coordinates change according to

$$\delta_G z^i = -g\alpha^\Lambda k_\Lambda^i(z) \ . \tag{2.7}$$

To close the gauge algebra on the scalars, the Killing vector fields must span a Lie-algebra with commutation relations

$$[k_\Lambda, k_\Sigma] = f_{\Lambda\Sigma}{}^\Pi k_\Pi \,, \tag{2.8}$$

and structure constants $f_{\Lambda\Sigma}{}^\Pi$ of some Lie-group G that one wishes to gauge. Not all holomorphic isometries can be gauged within $N = 2$ supergravity. The induced change on the sections needs to be consistent with the symplectic structure of the theory, and this requires the holomorphic sections to transform as

$$\delta_G \begin{pmatrix} X^\Lambda \\ F_\Lambda \end{pmatrix} = -g\alpha^\Sigma \left[T_\Sigma \cdot \begin{pmatrix} X^\Lambda \\ F_\Lambda \end{pmatrix} + r_\Sigma(z) \begin{pmatrix} X^\Lambda \\ F_\Lambda \end{pmatrix} \right] \ . \tag{2.9}$$

The second term induces a Kähler transformation on the Kähler potential

$$\delta_G \mathcal{K}(z,\bar z) = g\alpha^\Lambda (r_\Lambda(z) + \bar r_\Lambda(\bar z)) \,, \tag{2.10}$$

for some holomorphic functions $r_\Lambda(z)$. The first term in (2.9) contains a constant matrix T_Σ that acts on the sections as infinitesimal symplectic transformations. For electric gaugings, which we consider in this section, we mean, by definition, that the representation is of the form

$$T_\Lambda = \begin{pmatrix} -f_\Lambda & 0 \\ c_\Lambda & f_\Lambda^t \end{pmatrix} \,, \tag{2.11}$$

where f_Λ denotes the matrix $(f_\Lambda)_\Sigma{}^\Pi = f_{\Lambda\Sigma}{}^\Pi$ and f_Λ^t is the transposed. The tensor $c_{\Lambda,\Sigma\Pi} \equiv (c_\Lambda)_{\Sigma\Pi}$ is required to be symmetric for T_Λ to be a symplectic generator. Moreover, there are some additional constraints on the c_Λ in order for the T_Λ to be symplectically embedded within the same Lie-algebra as in (2.8). One can easily derive them, for explicit

[2]We will assume in the remainder of the paper that the gauge coupling constant $g \neq 0$. The case of $g = 0$ is treated in the literature in e.g. [24].

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formulae see [9], or (3.5). Finally, closure of the gauge transformations on the Kähler potential requires that

$$k_\Lambda^i \partial_i r_\Sigma - k_\Sigma^i \partial_i r_\Lambda = f_{\Lambda\Sigma}{}^{\Pi} r_\Pi \,. \tag{2.12}$$

We summarize some other important identities on vector multiplet gauging in appendix B.

Magnetic gaugings allow also non-zero entries in the upper-right corner of T_Λ, but we will not consider them here. The gauged action, in particular the scalar potential, that we consider below is not invariant under magnetic gauge transformation. To restore this invariance, one needs to introduce massive tensor multiplets, but the most general lagrangian with both electric and magnetic gauging is not fully understood yet (for some partial results see [27–30]).

Given a choice for the gauge group (2.11), one can reverse the order of logic and determine the form of the Killing vectors, and therefore the gauge transformations of the scalar fields z^i. This analysis was done in [31], and the result is written in the appendix, see (B.6).

We now return to the BPS conditions. The second and third term in the supersymmetry variation of the gauginos, equation (2.4), need also to vanish separately, since they multiply independent spinors of the same chirality. For the second term, this leads to

$$G_{\mu\nu}^{i\,-} \equiv -g^{i\bar{\jmath}} \bar{f}_{\bar{\jmath}}^{\Lambda} (\mathrm{Im}\mathcal{N}_{\Lambda\Sigma}) F_{\mu\nu}^{\Sigma\,-} = 0 \,, \tag{2.13}$$

where $g^{i\bar{\jmath}}$ is the inverse Kähler metric with Kähler potential $\mathcal{K}$ from (2.1), and

$$\overline{\mathcal{N}}_{\Lambda\Sigma} \equiv \begin{pmatrix} D_i F_\Lambda \\ \bar{F}_\Lambda \end{pmatrix} \cdot \begin{pmatrix} D_i X^\Sigma \\ \bar{X}^\Sigma \end{pmatrix}^{-1} , \qquad f_i^\Lambda \equiv \mathrm{e}^{\mathcal{K}/2} D_i X^\Lambda \,, \tag{2.14}$$

with $D_i X^\Lambda = (\partial_i + \mathcal{K}_i) X^\Lambda$ and similarly $D_i F_\Lambda = (\partial_i + \mathcal{K}_i) F_\Lambda$. The anti-selfdual part of any real two-form $T_{\mu\nu}$ is denoted by $T^-_{\mu\nu}$, and complex conjugation gives the selfdual part, see the appendix of [11].

Finally, setting the third term in the supersymmetry variation to zero leads to

$$W^{iAB} \equiv k_\Lambda^i \bar{L}^\Lambda \epsilon^{AB} + i g^{i\bar{\jmath}} \bar{f}_{\bar{\jmath}}^{\Lambda} P_\Lambda^x \sigma_x^{AB} = 0 \,, \tag{2.15}$$

where $L^\Lambda = \mathrm{e}^{\mathcal{K}/2} X^\Lambda$ (in analogy, $M_\Lambda \equiv \mathrm{e}^{\mathcal{K}/2} F_\Lambda$) and P_Λ^x are the triplet of moment maps associated with the Killing vector fields $\tilde{k}_\Lambda$ on the quaternionic geometry.[3] These Killing vectors are used to determine the gauge transformations of the hypermultiplet scalars under the gauge group. The only requirement is that the Killing equation is satisfied, i.e. they are isometries on the quaternion-Kähler manifold, and they satisfy the same Lie-bracket as in (2.8). Of course, a given quaternion-Kähler manifold can allow inequivalent choices of Killing vectors with the same Lie-algebra. These choices lead to different models with different physics. One obvious choice is to set all the Killing vectors to zero, and so all hypermultiplet scalars remain neutral under the gauge group. The gauging then remains solely active on the vector multiplet scalars.

[3] For the explicit relation between moment maps and Killing vectors in the quaternionic case, as well as other useful identities in the hypermultiplet sector, see the standard references.

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Close inspection of (2.15) shows that both terms are linearly independent in $\mathrm{SU}(2)_R$ space, hence they must vanish separately,

$$k^i_\Lambda \bar{L}^\Lambda = 0\,, \qquad P^x_\Lambda f^\Lambda_i = 0\,, \tag{2.16}$$

and their complex conjugates.

2.2 Hyperinos

The fields in the hypermultiplet sector comprise $4n_H$ scalars q^u, and $2n_H$ positive chirality fermions ζ_α and their complex conjugates $(\zeta_\alpha)^* = \mathbb{C}_{\alpha\beta}\zeta^\beta$ with negative chirality. Under $N = 2$ local supersymmetry, these hyperinos transform as

$$\delta_\varepsilon \zeta_\alpha = i\,\mathcal{U}^{B\beta}_u \nabla_\mu q^u \gamma^\mu \varepsilon^A \epsilon_{AB} \mathbb{C}_{\alpha\beta} + g N^A_\alpha \varepsilon_A\,, \tag{2.17}$$

again, up to terms that are of higher order in the fermions. The hyperino mass matrix N^A_α is defined by

$$N^A_\alpha \equiv 2\,\mathcal{U}^A_{\alpha\,u} \tilde{k}^u_\Lambda \bar{L}^\Lambda\,, \tag{2.18}$$

with L^Λ as given just below (2.15).

Similarly as for the gauginos, $N = 2$ supersymmetric configurations require the two terms in (2.17) to vanish separately. Since the quaternionic vielbeine are invertible and nowhere vanishing, the scalars need to be covariantly constant,

$$\nabla_\mu q^u \equiv \partial_\mu q^u + g A^\Lambda_\mu \tilde{k}^u_\Lambda = 0\,, \tag{2.19}$$

implying the integrability conditions

$$F^\Lambda_{\mu\nu}\, \tilde{k}^u_\Lambda = 0\ . \tag{2.20}$$

Furthermore, there is a second condition from (2.17) coming from the vanishing of the hyperino mass matrix N^A_α. This leads to

$$\tilde{k}^u_\Lambda L^\Lambda = 0\,, \tag{2.21}$$

and complex conjugate.

In the absence of hypermultiplets, i.e. when $n_H = 0$, the $N = 2$ conditions from the variations of the hyperinos disappear. However, the second condition in (2.16) remains, with the moment maps replaced by FI parameters.[4] So our formalism automatically includes the case $n_H = 0$.

[4]In the absence of any hypermultiplets the quantities P^x_Λ need not vanish. Instead, they can be constants, which can be non-vanishing for gauge groups SU(2) or U(1). These constants are sometimes referred to as Fayet-Illiopoulos (FI) terms. See e.g. [32] for a discussion.

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2.3 Gravitinos

The fermions in the gravitational sector are two gravitinos of opposite chirality $\psi_{\mu A}$ and its complex conjugate $\psi_\mu^A = (\psi_{\mu A})^*$. In gauged supergravity, their supersymmetry transformation rules are (up to irrelevant higher order terms in the fermions)

$$\delta_\varepsilon \psi_{\mu A} = \nabla_\mu \varepsilon_A + T^-_{\mu\nu}\gamma^\nu \epsilon_{AB}\varepsilon^B + igS_{AB}\gamma_\mu \varepsilon^B \ . \tag{2.22}$$

Here, $\nabla_\mu \varepsilon_A$ is the gauged supercovariant derivative (specified below), and

$$T^-_{\mu\nu} \equiv 2iF^{\Lambda\,-}_{\mu\nu}(\mathrm{Im}\mathcal{N}_{\Lambda\Sigma})L^\Sigma \, , \qquad S_{AB} \equiv \frac{i}{2}(\sigma_x)_{AB}P^x_\Lambda L^\Lambda \ . \tag{2.23}$$

The matrices $T_{\mu\nu}$ and S_{AB} are called the graviphoton field strength and the gravitino mass-matrix respectively. Notice again that for $n_H = 0$, in fact even also in the absence of vector multiplets when $n_V = 0$, the gravitino mass-matrix can be non-vanishing and constant. In the Lagrangian, which we discuss in the next section, this leads to a (negative) cosmological constant term. The anti-selfdual part of the graviphoton field strength $T_{\mu\nu}$ satisfies the identity

$$F^{\Lambda\,-}_{\mu\nu} = i\bar{L}^\Lambda T^-_{\mu\nu} + 2f_i^\Lambda G^{i\,-}_{\mu\nu} \, , \tag{2.24}$$

with $G^{i\,-}_{\mu\nu}$ defined in (2.13). From the vanishing of the gaugino variation, we have that $G^{i\,-}_{\mu\nu} = 0$, and hence a maximally supersymmetric configuration must satisfy $F^{\Lambda\,-}_{\mu\nu} = i\bar{L}^\Lambda T^-_{\mu\nu}$, or

$$F^\Lambda_{\mu\nu} = i\overline{L}^\Lambda T^-_{\mu\nu} - iL^\Lambda T^+_{\mu\nu} \ . \tag{2.25}$$

Using this, we then see that equation (2.21) implies the integrability conditions (2.20) in the hypermultiplet sector. For the integrability equations in the vector multiplet sector, the situation is more subtle, as the Killing vectors are complex and holomorphic. Now, the BPS condition (2.16) only implies that

$$k^i_\Lambda F^\Lambda_{\mu\nu} = -ik^i_\Lambda L^\Lambda T^+_{\mu\nu} \ .$$

As a consequence, the integrability condition (2.6) is only guaranteed when $k^i_\Lambda L^\Lambda = 0$ (or, when $T_{\mu\nu} = 0$, but then all the field strengths are zero). So, for $T_{\mu\nu} \neq 0$, a necessary condition for a maximally supersymmetric configuration is that $k^i_\Lambda L^\Lambda = 0$. Furthermore, in appendix B we prove that

$$k^i_\Lambda L^\Lambda = 0 \quad \Leftrightarrow \quad P_\Lambda L^\Lambda = 0 \tag{2.26}$$

where P_Λ is the special Kähler moment map, defined in (B.1). In terms of (B.5), one sees that this condition involves both the structure constants and the matrix c_Λ. Hence the integrability condition is satisfied for those configurations satisfying $P_\Lambda L^\Lambda = 0$. The integrability condition might only locally be sufficient, but this fine for our purposes. One might however check in addition whether the covariant constancy of the vector multiplet scalars imposes further (global) restrictions.

To solve the constraints from the gravitino variation, we must first specify the gauged supercovariant derivative on the supersymmetry parameter. It can be written as

$$\nabla_\mu \varepsilon_A = \left(\partial_\mu - \frac{1}{4}\omega_\mu^{ab}\gamma_{ab}\right)\varepsilon_A + \frac{i}{2}A_\mu \varepsilon_A + \omega_{\mu\, A}{}^B \varepsilon_B \; . \tag{2.27}$$

The conventions for the spin connection, appearing between the brackets, are specified in the appendix. Furthermore, there appear two other connections associated to the special Kähler and quaternion-Kähler manifolds. We need to compute their curvatures since they enter the integrability conditions that follow from the Killing spinor equations. The first one is called the gauged U(1) Kähler-connection, defined by [11, 31]

$$A_\mu \equiv -\frac{i}{2}\Big(\partial_i \mathcal{K} \nabla_\mu z^i - \partial_{\bar{\imath}} \mathcal{K} \nabla_\mu \bar{z}^{\bar{\imath}}\Big) + \frac{i}{2} g A_\mu^\Lambda (r_\Lambda - \bar{r}_\Lambda) \; . \tag{2.28}$$

Under a gauge transformation, one finds that

$$\delta_G A_\mu = \frac{i}{2} g\, \partial_\mu \Big[\alpha^\Lambda (r_\Lambda - \bar{r}_\Lambda)\Big] \; . \tag{2.29}$$

The curvature of this connection can be computed to be

$$F_{\mu\nu} = 2i g_{i\bar{\jmath}} \nabla_{[\mu} z^i \nabla_{\nu]} \bar{z}^{\bar{\jmath}} - g F_{\mu\nu}^\Lambda P_\Lambda \, , \tag{2.30}$$

where P_Λ is the moment map, defined in (B.1), and we have used the equivariance condition (B.3). For maximally supersymmetric configurations, the scalars are covariantly constant and hence the curvature of the Kähler connections satisfies $F_{\mu\nu} = -gF_{\mu\nu}^\Lambda P_\Lambda$.

The second connection appearing in the gravitino supersymmetry variation is the gauged Sp(1) connection of the quaternion-Kähler manifold. It reads

$$\omega_{\mu\, A}{}^B \equiv \partial_\mu q^u \omega_{u\, A}{}^B + g A_\mu^\Lambda P_{\Lambda\, A}{}^B \, , \tag{2.31}$$

where $\omega_{u\, A}{}^B$ is the (ungauged) Sp(1) connection of the quaternion-Kähler manifold, whose curvatures are related to the three quaternionic two-forms. The effect of the gauging is to add the second term on the right hand side of (2.31), proportional to the triplet of moment maps of the quaternionic isometries, with $P_{\Lambda\, A}{}^B = P_\Lambda^x (\sigma^x)_A{}^B$. The curvature of (2.31) can then be computed to be

$$\Omega_{\mu\nu\, A}{}^B = 2\Omega_{\mathrm{uv\, A}}{}^B \nabla_{[\mu} q^u \nabla_{\nu]} q^v + g F_{\mu\nu}^\Lambda P_{\Lambda\, A}{}^B \, , \tag{2.32}$$

where $\Omega_{\mathrm{uv\, A}}{}^B$ is the quaternionic curvature. For fully BPS solutions, we have $\Omega_{\mu\nu\, A}{}^B = gF_{\mu\nu}^\Lambda P_{\Lambda\, A}{}^B$.

We can now investigate the integrability conditions that follow from the vanishing of the gravitino transformation rules (2.22). From the definition of the supercovariant derivative (2.27), we find[5]

$$[\nabla_\mu, \nabla_\nu]\varepsilon_A = -\frac{1}{4} R_{\mu\nu}{}^{ab}\gamma_{ab}\, \varepsilon_A - \frac{i}{2} g F_{\mu\nu}^\Lambda P_\Lambda \varepsilon_A + g F_{\mu\nu}^\Lambda P_{\Lambda\, A}{}^B \varepsilon_B \, , \tag{2.33}$$

[5]Strictly speaking, we get the supercovariant curvatures appearing in (2.33), which also contain fermion bilinears. Since the fermions are zero on maximally supersymmetric configurations, only the bosonic part of the curvatures remains.

where we have used the covariant constancy of the scalars. We remind that P_Λ are the moment maps on the special Kähler geometry, whereas $P_{\Lambda\,A}{}^B$ are the quaternion-Kähler moment maps. Alternatively, we can compute the commutator from the vanishing of the gravitino variations spelled out in (2.22). By equating this to the result of (2.33), we get a set of constraints. Details of the calculation are given in appendix C, and the results can be summarized as follows. First of all, we find the covariant constancy of the graviphoton field strength[6]

$$D_\rho T^+_{\mu\nu} = 0 \,. \tag{2.34}$$

Secondly, we get that the quaternionic moment maps must satisfy

$$\epsilon^{xyz} P^y \overline{P}^z = 0 \,, \qquad P^x \equiv L^\Lambda P^x_\Lambda \,. \tag{2.35}$$

Moreover, there are cross terms between the graviphoton and the moment maps, which enforce the conditions

$$T^+_{\mu\nu} P^x = 0 \,. \tag{2.36}$$

This equation separates the classification of BPS configurations in two sectors, those with a solution of $P^x = 0$ at a particular point (or locus) in field space, and those with non-vanishing P^x (for at least one index x) but $T_{\mu\nu} = 0$. We will see later on that this distinction corresponds to zero or non-zero (and negative) cosmological constant in the spacetime.

Another requirement that follows from the gravitino integrability conditions is

$$F^\Lambda_{\mu\nu} P_\Lambda = 0 \,, \tag{2.37}$$

where P_Λ is defined in (B.1), and is real. Using (2.25), this is equivalent to the condition

$$\bar{L}^\Lambda P_\Lambda T^-_{\mu\nu} = L^\Lambda P_\Lambda T^+_{\mu\nu} \,. \tag{2.38}$$

Since anti-selfdual and selfdual tensors are linearly independent, it means that $P_\Lambda L^\Lambda = 0$ and complex conjugate (again, for $T_{\mu\nu} \neq 0$). This requirement is already imposed by the integrability conditions on the vector multiplet scalars, see (2.26), so (2.37) does not lead to any new constraint.

Finally, there is the condition on the spacetime Riemann curvature. It reads

$$R_{\mu\nu\rho\sigma} = 4 T^+_{\mu[\sigma} T^-_{\rho]\nu} + g^2 P^x \overline{P^x} g_{\mu\sigma} g_{\nu\rho} - (\mu \leftrightarrow \nu) \,. \tag{2.39}$$

It can be checked that this leads to a vanishing Weyl tensor, implying conformal flatness. From the curvature, we can compute the value of the Ricci-scalar to be

$$R = -12 g^2 P^x \overline{P^x} \,. \tag{2.40}$$

Hence, the classification of fully supersymmetric configurations separates into negative scalar curvature with $P^x \overline{P^x} \neq 0$, and zero curvature with $P^x = 0$ at the supersymmetric point. In both of these cases there are important simplifications.

[6] Recall that T^+ and T^- are related by complex conjugation, and hence the vanishing of DT^+ implies $DT = 0$.

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2.3.1 Negative scalar curvature

The case of negative scalar curvature is characterized by $T_{\mu\nu} = 0$ and $P^x\overline{P^x} \neq 0$ at the supersymmetric point. Since the BPS conditions imply that then both $T_{\mu\nu}$ and $G^{i-}_{\mu\nu} = 0$ (see equation (2.13)), we find that all field strengths should be zero: $F^\Lambda_{\mu\nu} = 0$. The gauge fields then are required to be pure gauge, but can still be topologically non-trivial. Furthermore, because of the vanishing field strengths, the integrability conditions on the scalar fields are satisfied, and a solution for the sections $X^\Lambda(z)$ is obtained by a gauge transformation on the constant (in spacetime) sections. Finally, the Riemann tensor is given by

$$R_{\mu\nu\rho\sigma} = g^2 P^x\overline{P^x}\left(g_{\mu\sigma}g_{\nu\rho} - g_{\nu\sigma}g_{\mu\rho}\right) .$$

which shows that the space is maximally symmetric, and therefore locally AdS_4. The scalar curvature is $R = -12g^2P^x\overline{P^x}$.

2.3.2 Zero scalar curvature

The class of zero curvature is characterized by configurations for which $P^x = 0$ at the supersymmetric point. In this case, we can combine the conditions $P^x_\Lambda f^\Lambda_i = 0$ and $P^x \equiv P^x_\Lambda L^\Lambda = 0$ into

$$P^x_\Lambda \begin{pmatrix} \bar{L}^\Lambda \\ f^\Lambda_i \end{pmatrix} = 0 .$$

The matrix appearing here, is the invertible matrix of special geometry (as used in (2.14)), hence we conclude that $P^x_\Lambda = 0$. The Riemann tensor is then

$$R_{\mu\nu\rho\sigma} = 4T^+_{\mu[\sigma}T^-_{\rho]\nu} - (\mu \leftrightarrow \nu) .$$

From the covariant constancy of the graviphoton, condition (2.34), we find $D_\rho R_{\mu\nu\sigma\tau} = 0$. Spaces with covariantly constant Riemann tensor are called locally symmetric, and they are classified, see e.g. [23, 26, 33]. In our case we also have zero scalar curvature, and then only three spaces are possible:

1. Minkowski space M_4 ($T_{\mu\nu} = 0$)
2. $AdS_2 \times S^2$
3. The pp-wave solution

The explicit metrics and field strengths for the latter two cases (M_4 and AdS_4 are well-known and have vanishing field strengths) are listed in appendix D.

2.4 Summary

Let us now summarize the results. There are two different classes: negative scalar curvature (leading to AdS_4) and zero scalar curvature solutions (leading to M_4, $AdS_2 \times S^2$ or the pp-wave).

The result of our analysis is that all the conditions on the spacetime dependent part are explicitly solved,[7] and the remaining conditions are purely algebraic, and depend only on the geometry of the special Kähler and quaternionic manifolds. The solutions to these algebraic equations define the configuration space of maximally supersymmetric configurations. There are two separate cases:

2.4.1 Negative scalar curvature (AdS_4)

This case is characterized by configurations for which $P^x\overline{P^x} \neq 0$ at the supersymmetric point. The BPS conditions are

$$\begin{aligned} k^i_\Lambda \overline{L}^\Lambda &= 0 & \qquad \tilde{k}^u_\Lambda L^\Lambda &= 0 \\ P^x_\Lambda f^\Lambda_i &= 0 & \qquad \epsilon^{xyz} P^y \overline{P^z} &= 0\,, \end{aligned}$$

which should be satisfied at a point (or a locus) in field space. The field strengths are zero, $F^\Lambda_{\mu\nu} = 0$, and the space-time is AdS_4 with scalar curvature $R = -12g^2 P^x\overline{P^x}$.

2.4.2 Zero scalar curvature (M_4, $AdS_2 \times S^2$ or pp-wave)

In this case, the BPS conditions are

$$\begin{aligned} k^i_\Lambda \overline{L}^\Lambda &= 0 & \qquad \tilde{k}^u_\Lambda L^\Lambda &= 0 \\ P_\Lambda L^\Lambda &= 0 & \qquad P^x_\Lambda &= 0\,. \end{aligned}$$

We remind that, when $T_{\mu\nu} = 0$ (Minkowski space), all field strengths are vanishing ($F^\Lambda_{\mu\nu} = 0$), and the condition $P_\Lambda L^\Lambda = 0$ need not be satisfied. For non-vanishing $T_{\mu\nu}$, the field strengths are given by (2.25), and using formula (B.5) the condition $P_\Lambda L^\Lambda = 0$ is equivalent to

$$L^\Lambda \bar{L}^\Pi f_{\Lambda\Pi}{}^\Sigma M_\Sigma + L^\Lambda L^\Pi c_{\Lambda,\Pi\Sigma} \bar{L}^\Sigma = 0\,, \tag{2.41}$$

where we remind that $M_\Lambda \equiv \mathrm{e}^{\mathcal{K}/2} F_\Lambda$. Hence the existence of maximal BPS configurations also depends on the c_Λ-matrix characterizing the Chern-Simons-like terms.

3 Lagrangians and scalar potentials

Since all fermions are equal to zero for $N = 2$ supersymmetric configurations, we can concentrate on the bosonic part of the Lagrangian, with action $S = \int \mathrm{d}^4x \sqrt{g}\mathcal{L}$. It can be read off from [9, 11],

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2}R(g) + g_{i\bar{\jmath}}\nabla^\mu z^i \nabla_\mu \bar{z}^{\bar{\jmath}} + h_{\rm uv}\nabla^\mu q^u \nabla_\mu q^v + (\mathrm{Im}\,\mathcal{N}_{\Lambda\Sigma})F^\Lambda_{\mu\nu}F^{\Sigma\,\mu\nu} \\ & + \frac{1}{2}(\mathrm{Re}\,\mathcal{N}_{\Lambda\Sigma})\epsilon^{\mu\nu\rho\sigma}F^\Lambda_{\mu\nu}F^\Sigma_{\rho\sigma} - \frac{4}{3}g\, c_{\Lambda,\Sigma\Pi}\,\epsilon^{\mu\nu\rho\sigma}A^\Lambda_\mu A^\Sigma_\nu \left(\partial_\rho A^\Pi_\sigma - \frac{3}{8} f_{\Omega\Gamma}{}^\Pi A^\Omega_\rho A^\Gamma_\sigma\right) - V(z,\bar{z},q)\,, \end{aligned} \tag{3.1}$$

[7] This is apart from the scalar fields and Killing spinors, which are spacetime dependent. The integrability conditions that we have imposed guarantee locally the existence of a solution, although we did not explicitly construct it. Its construction cannot be done in closed form in full generality, but can be worked out in any given example [23].

with scalar potential

$$V = g^2 \Big[(g_{i\bar{\jmath}} k^i_\Lambda k^{\bar{\jmath}}_\Sigma + 4 h_{\mathrm{uv}} k^u_\Lambda k^v_\Sigma) \bar{L}^\Lambda L^\Sigma + (g^{i\bar{\jmath}} f^\Lambda_i \bar{f}^\Sigma_{\bar{\jmath}} - 3 \bar{L}^\Lambda L^\Sigma) P^x_\Lambda P^x_\Sigma \Big] \ . \tag{3.2}$$

The Chern-Simons-like term on the second line of (3.1) can be determined from the non gauge-invariance of the period matrix. From (2.14) one finds

$$\delta_G \mathcal{N}_{\Lambda\Sigma} = -g\alpha^\Pi \left(f_{\Pi\Lambda}{}^\Gamma \mathcal{N}_{\Gamma\Sigma} + f_{\Pi\Sigma}{}^\Gamma \mathcal{N}_{\Gamma\Lambda} + c_{\Pi,\Lambda\Sigma} \right) . \tag{3.3}$$

Since the right hand side is real, only the topological term proportional to $\mathrm{Re}\mathcal{N}_{\Lambda\Sigma}$ in the action transforms. This transformation is compensated by the gauge transformation of the other terms in the second line, using the various constraints on the (symmetric) c_Λ. In the abelian case, the only constraint is that the totally symmetrized c-tensor vanishes, i.e.

$$c_{\Lambda,\Sigma\Pi} + c_{\Pi,\Lambda\Sigma} + c_{\Sigma,\Pi\Lambda} = 0 \ . \tag{3.4}$$

This implies that for a single vector field, the Chern-Simons-like term vanishes. The additional constraints for nonabelian gaugings involve the structure constants [9]:

$$f_{\Lambda\Sigma}{}^\Gamma c_{\Gamma,\Pi\Omega} + f_{\Omega\Sigma}{}^\Gamma c_{\Lambda,\Gamma\Pi} + f_{\Pi\Sigma}{}^\Gamma c_{\Lambda,\Gamma\Omega} + f_{\Lambda\Omega}{}^\Gamma c_{\Sigma,\Gamma\Pi} + f_{\Lambda\Pi}{}^\Gamma c_{\Sigma,\Gamma\Omega} = 0 \ . \tag{3.5}$$

The scalar potential can be written in terms of the mass-matrices,

$$V = -6 S^{AB} S_{AB} + \frac{1}{2} g_{i\bar{\jmath}} W^{iAB} \overline{W}^{\bar{\jmath}}_{AB} + N^A_\alpha N^\alpha_A \ . \tag{3.6}$$

Since the gaugino and hyperino mass-matrices, W^{iAB} and N^A_α respectively, vanish on $N = 2$ supersymmetric configurations, one sees that the scalar potential is semi-negative definite, and determined by the gravitino mass-matrix S_{AB}. Even in the absence of vector and hypermultiplets, the gravitino mass-matrix can be non-vanishing, leading to a negative cosmological constant in the Lagrangian. Using (3.2), we find for $N = 2$ preserving configurations

$$V = -3 g^2 \bar{L}^\Lambda L^\Sigma P^x_\Lambda P^x_\Sigma \ . \tag{3.7}$$

In the absence of hypermultiplets, $N = 2$ preserving AdS_4 vacua can therefore only be generated by non-trivial Fayet-Illiopoulos terms.

It can be verified that maximally supersymmetric configurations also solve the equations of motion. To show this, one varies the lagrangian (3.1) and uses the identities (3.4), (3.5) and the formulas in section 2.4. After a somewhat tedious but straightforward computation one sees that all equations of motion are indeed satisfied by the maximally supersymmetric configurations.

4 Examples

In this section we list some (string theory motivated) examples of $N = 2, D = 4$ theories, leading to $N = 2$ supersymmetric configurations. We will first mention briefly some already known and relatively well-understood $N = 2$ vacua from string theory and then concentrate

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on our two main examples in subsections 4.1 and 4.2 that exhibit best the different features discussed above. In the last subsection we include some supergravity models, not necessarily obtained from string compactifications, leading to AdS_4 vacua that can be of interest.

Obtaining gauged $N = 2, D = 4$ supergravity seems to be important for string theory compactifications since it is an intermediate step between the more realistic $N = 1$ models and the mathematically controllable theories. Thus in the last decade there has been much literature on the subject. An incomplete list of examples consists of [15, 18–21] and it is straightforward to impose and solve the maximal supersymmetry constraints in each case. In some cases the vacua have been already discussed or must exist from general string theory/M-theory considerations.

For example, it was found that the coset compactifications studied in [20] do not lead to $N = 2$ supersymmetric configurations. This can also be seen from imposing the constraints in section 2.4. In contrast, the compactification on $K3\times T^2/\mathbb{Z}_2$ presented in [15] does exhibit $N = 2$ solutions with non-trivial hypermultiplet gaugings. The authors of [15] explicitly found $N = 2$ Minkowski vacua by satisfying the same susy conditions as in section 2.4. From our analysis, it trivially follows that also the pp-wave and the $AdS_2\times S^2$ backgrounds are maximally supersymmetric. To check this, one only needs to verify (2.41), and this is satisfied due to a vanishing c-tensor and the abelian gauging in the hypermultiplet sector.

A similar example is provided by the (twisted) $K3\times T^2$ compactification of the heterotic string, recently analyzed in [21]. For abelian gaugings, one can verify that the three zero scalar curvature vacua are present in these models.

We now turn to discuss the remaining models in more detail.

4.1 M-theory compactification on SU(3) structure manifolds

There is a very interesting model for $N = 2, D = 4$ supergravity with non-abelian gauging of the vector multiplet sector and non-trivial c-tensor, arising from compactifications of M-theory on seven-manifolds with SU(3) structure [18] (more precisely, they consider Calabi-Yau (CY) threefolds fibered over a circle). For the precise M-theory set-up, we refer the reader to [18]; here we only discuss the relevant data for analyzing the maximal supersymmetry conditions:

- the vector multiplet space can be parametrized by special coordinates, $X^\Lambda = (1, t^i = b^i + iv^i)$ and prepotential

$$F(X) = -\frac{1}{6}\kappa_{ijk}\frac{X^iX^jX^k}{X^0}\,, \tag{4.1}$$

  with the well-known triple intersection numbers κ_{ijk} that depend on the particular choice of the CY-manifold. This gives the Kähler potential

$$\mathcal{K} = -\log\left[\frac{i}{6}\kappa_{ijk}(t^i-\bar{t}^i)(t^j-\bar{t}^j)(t^k-\bar{t}^k)\right] \equiv -\log \mathrm{Vol}\,, \tag{4.2}$$

  where Vol denotes the volume of the compact manifold. The gauge group is non-abelian with structure constants

$$f_{\Lambda\Sigma}{}^0 = 0 = f_{ij}{}^k, \qquad f_{i0}{}^j = -M_i^j\,, \tag{4.3}$$

and a c-tensor whose only non-vanishing components are

$$c_{i,jk} = \frac{1}{2} M_i^l \kappa_{ljk} \; . \tag{4.4}$$

The constant matrix M_i^j specifies the Killing vectors and moment-maps of the special Kähler manifold:

$$k_0^j = -M_k^j t^k \, , \qquad\qquad k_i^j = M_i^j \, , \tag{4.5}$$

and

$$P_0 = -M_i^j t^i \partial_j \mathcal{K} \, , \qquad\qquad P_i = M_i^j \partial_j \mathcal{K} \; . \tag{4.6}$$

Not for any choice of M_i^j is the Killing equation satisfied. As explained in [18], this is only the case when the relation (3.4) holds. This also ensures that (3.5) is satisfied, as one can easily check.

- generally in this class of compactifications there always appear hypermultiplet scalars, but there is no gauging of this sector, so the Killing vectors and the moment maps P^x_Λ are vanishing.

The scalar potential in this case reduces to the simple formula

$$V = -\frac{8}{\mathrm{Vol}^2} M_i^k M_j^l \kappa_{klm} v^i v^j v^m \, , \tag{4.7}$$

which is positive semi-definite.

Analyzing the susy conditions is rather straightforward. Since $P^x = 0$, the only allowed $N = 2$ vacua are the ones with zero-scalar curvature. What is left for us to check are the conditions $k^i_\Lambda \bar{L}^\Lambda = 0$ and $P_\Lambda L^\Lambda = 0$. The latter is very easy to check and holds as an identity at every point in the special Kähler manifold. Also, it is equivalent to the relation $k^i_\Lambda L^\Lambda = 0$ which is satisfied whenever there exists a prepotential [10]. The condition $k^i_\Lambda \bar{L}^\Lambda = 0$ eventually leads to

$$\frac{M_j^i (t^j - \bar{t}^j)}{\mathrm{Vol}} = 2i \frac{M_j^i v^j}{\mathrm{Vol}} = 0 \, , \qquad \forall i \; . \tag{4.8}$$

The solution to the above equation that always exists is the decompactification limit when $\mathrm{Vol} \to \infty$. The other more interesting solutions depend on the explicit form of the matrix M. In case M_i^j is invertible there are no further solutions to (4.8). On the other hand, when M has zero eigenvalues we can have $N = 2$ M-theory vacua, given by (a linear combination of) the corresponding zero eigenvectors of M. For the supergravity approximation to hold, one might require that this solution leads to a non-vanishing (and large) volume of the CY. Each eigenvector will correspond to a flat direction of the scalar potential, and with $V = 0$ along these directions. The case where the full matrix M is zero corresponds to a completely flat potential, the one of a standard M-theory compactification on $CY \times S^1$ without gauging.

Thus it is clear that M_i^j is an important object for this type of M-theory compactifications and we now give a few more details on its geometrical meaning [18]. In the above class of M-theory compactifications we have a very specific fibration of the Calabi-Yau manifold over the circle. It is chosen such that only the second cohomology $H^{(1,1)}(CY_3)$ is twisted with respect to the circle, while the third cohomology $H^3(CY_3)$ is unaffected. Thus the hypermultiplet sector remains ungauged as in regular $CY_3 \times S^1$ compactification, while the vector multiplets feel the twisting and are gauged. This twisting is parametrized exactly by the matrix M, as it determines the differential relations of the harmonic (on the CY_3) two-forms:

$$\mathrm{d}\omega_i = M_i^j \omega_j \wedge \mathrm{d}z \,, \tag{4.9}$$

where z is the circle coordinate.

Let us now zoom in on the interesting case when we have nontrivial zero eigenvectors of M, corresponding to non-vanishing volume of the CY. For a vanishing volume, or a vanishing two-cycle, the effective supergravity description might break down due to additional massless modes appearing in string theory.[8] Therefore the really consistent and relevant examples for $N = 2$ vacua are only those when the matrix M is non-invertible with corresponding zero eigenvectors that give nonzero value for every v^i.

To illustrate this better, we consider a particular example, given in section 2.5 of [18], of a compactification where the CY_3 is a $K3$-fibration. In this setting one can explicitly construct an M-matrix, compatible with the intersection numbers κ_{ijk}. Here one can find many explicit cases where all of the above described scenarios happen. As a very simple and suggestive example we consider the 5-scalar case with $\kappa_{123} = -1, \kappa_{144} = \kappa_{155} = 2$, and twist-matrix

$$M = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & -2 & -2 \\ 0 & 0 & -4 & 2 & 2 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{pmatrix} . \tag{4.10}$$

The general solution of $M \cdot \vec{v} = 0$ is

$$\vec{v} = \lambda \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \\ 1 \\ 2 \\ 0 \end{pmatrix} + \nu \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 2 \end{pmatrix} , \tag{4.11}$$

and the resulting volume is

$$\mathrm{Vol} = 8\lambda \left(2\mu^2 + 2\nu^2 + (\mu - \nu)^2 \right) , \tag{4.12}$$

which is clearly positive semi-definite. In the case when either μ or ν vanishes we have a singular manifold that is still a solution to the maximal supersymmetry conditions. When all three coefficients (that are essentially the remaining unstabilized moduli fields) are non-zero, we have a completely proper solution both from supergravity and string theory point of view, thus providing an example of SU(3) structure compactifications

[8] For a detailed analysis of the possibilities in a completely analogous case in five dimensions see [34]

with zero-curvature $N = 2$ vacua. This example can be straightforwardly generalized to a higher number of vector multiplets, as well as to the lower number of 4 scalars (there cannot be less than 4 vector multiplets in this particular case).

It is interesting to note in passing that a special case of the general setup described above was already known for more than twenty years in [9] (3.21), where $M_1^1 = -2, M_2^2 = 1$, and $\kappa_{122} = 2$. It was derived purely from 4d supergravity considerations, but it now seems that one can embed it in string theory.

4.2 Reduction of M-theory on Sasaki-Einstein$_7$

There has been much advance in the last years in understanding Sasaki-Einstein manifolds and their relevance for M-theory compactifications, both from mathematical and physical perspective. These spaces are good candidates for examples of the AdS_4/CFT_3 correspondence and an explicit reduction to $D = 4$ has been recently obtained in [19]. Originally the effective lagrangian includes magnetic gauging and a scalar-tensor multiplet, but after a symplectic rotation it can be formulated in the standard $N = 2$ formalism discussed here. After the dualization of the original tensor to a scalar we have the following data for the multiplets, needed for finding maximally supersymmetric vacua:

- there is one vector multiplet, given by $X^\Lambda = (1, \tau^2)$ and $F(X) = \sqrt{X^0(X^1)^3}$, leading to $F_\Lambda = (\frac{1}{2}\tau^3, \frac{3}{2}\tau^2)$ and Kähler potential

$$\mathcal{K} = -\log \frac{i}{2}(\tau - \bar{\tau})^3 \ . \tag{4.13}$$

  There is no gauging in this sector, i.e. $k_\Lambda^i = 0$ and $P_\Lambda = 0$ for all i, Λ. This also means that both $f_{\Lambda\Sigma}{}^\Pi$ and $c_{\Lambda,\Sigma\Pi}$ vanish.

- the hypermultiplet scalars are $\{\rho, \sigma, \xi, \bar{\xi}\}$ (ρ and σ are real, and ξ is complex) with the universal hypermultiplet metric:

$$\mathrm{d}s^2 = \frac{1}{4\rho^2}\mathrm{d}\rho^2 + \frac{1}{4\rho^2}\left(\mathrm{d}\sigma - i(\xi \mathrm{d}\bar{\xi} - \bar{\xi}\mathrm{d}\xi)\right)^2 + \frac{1}{\rho}\mathrm{d}\xi \mathrm{d}\bar{\xi} \ . \tag{4.14}$$

  We have an abelian gauging, given by (as there are no Killing vectors in the vector multiplet sector, we drop the tilde on the Killing vector fields in the hypermultiplet sector):

$$\tilde{k}_0 = 24\partial_\sigma - 4i(\xi\partial_\xi - \bar{\xi}\partial_{\bar{\xi}}) \,, \qquad \tilde{k}_1 = 24\partial_\sigma \,, \tag{4.15}$$

  and the moment maps, calculated in [19], are

$$\begin{aligned} &P_0^1 = -4\rho^{-1/2}(\xi + \bar{\xi}) \,, \quad &&P_0^2 = 4i\rho^{-1/2}(\xi - \bar{\xi}) \,, \quad &&P_0^3 = -\frac{12}{\rho} + 4\left(1 - \frac{\xi\bar{\xi}}{\rho}\right) \,, \\ &P_1^1 = 0, &&P_1^2 = 0 \,, &&P_1^3 = -\frac{12}{\rho} \ . \end{aligned} \tag{4.16}$$

We can now proceed to solving the maximal supersymmetry constraints. The conditions involving vector multiplet gauging are satisfied trivially, while from $\tilde{k}^u_\Lambda L^\Lambda = 0$ we obtain the conditions $\xi = \bar{\xi} = 0$ and $1+\tau^2 = 0$. Therefore $\tau = i$ (the solution $\tau = -i$ makes the Kähler potential ill-defined) and $\mathcal{K} = -\log 4$. However, not all the moment maps at this vacuum can be zero simultaneously, leaving AdS_4 as the only possibility for a $N=2$ vacuum solution. One can then see that $\epsilon_{xyz} P^y \overline{P^z} = 0$ is satisfied, so the only remaining condition is $P^3_\Lambda f^\Lambda_\tau = 0$. This fixes $\rho = 4$. Therefore we have stabilized all (ungauged) directions in moduli space: $\xi = \bar{\xi} = 0, \tau = i, \rho = 4$. The potential is nonzero in this vacuum since $P^3 = 2$, which means the only possibility for the space-time is to be AdS_4 with vanishing field strengths. This is indeed expected since SE_7 compactifications of M-theory lead to an $N=2$ AdS_4 vacuum, the one just described by us in the dimensionally reduced theory.

One can verify that this vacuum is stable under deformations in the hypermultiplet sector of the type discussed in [35, 36]. To show this, first observe that the condition $\tilde{k}^u_\Lambda L^\Lambda = 0$ for $u = \xi$ always ensures vanishing ξ. Secondly, one may verify that the deformations to the quaternionic moment maps are proportional to ξ, and hence the remaining $N=2$ conditions from section 2.4.1 are satisfied. It would be interesting to understand if this deformation corresponds to a perturbative one-loop correction in this particular type of M-theory compactification.

4.3 Other gaugings exhibiting AdS_4 vacua

Another example of an AdS_4 supersymmetric vacuum can be obtained from the universal hypermultiplet. In the same coordinates $\{\rho, \xi, \bar{\xi}, \sigma\}$ as used in the previous example, the metric is again given by (4.14). This space has a rotational isometry acting on ξ and $\bar{\xi}$, given by $\tilde{k}_1 - \tilde{k}_0$ in the notation of (4.15). We leave the vector multiplet sector unspecified for the moment, and gauge the rotation isometry by a linear combination of the gauge fields A^Λ_μ. This can be done by writing the Killing vector as

$$\tilde{k}^u_\Lambda = \alpha_\Lambda (0, i\xi, -i\bar{\xi}, 0)\,,$$

for some real constant parameters α_Λ. The quaternionic moment maps can be computed to be

$$P^x_\Lambda = \alpha_\Lambda \left(\frac{\xi + \bar{\xi}}{\sqrt{\rho}}, \frac{i(\xi - \bar{\xi})}{\sqrt{\rho}}, 1 - \frac{\xi\bar{\xi}}{\rho} \right)\,.$$

It can be seen that there are no points for which $P^x_\Lambda = 0, \forall x$, so this means that only AdS_4 $N=2$ vacua are possible. To complete the example, we have to specify the vector multiplet space, and solve the conditions $P^x_\Lambda f^\Lambda_i = 0$ and $\tilde{k}^u_\Lambda L^\Lambda = 0$. The latter can be solved as $\xi = \bar{\xi} = 0$, and then also $\epsilon^{xyz} P^y \overline{P^z} = 0$. The first one then reduces to $\alpha_\Lambda f^\Lambda_i = 0$. This condition is trivially satisfied when e.g. $n_V = 0$. A more complicated example is to take the special Kähler space of the previous subsection with no gauging in the vector multiplet sector. There is one complex scalar τ, a section $X^\Lambda = (1, \tau^2)$ and a prepotential $F = \sqrt{X^0 (X^1)^3}$. We then find a solution for $\tau = i\sqrt{\frac{-3\alpha_0}{\alpha_1}}$, under the condition that α_0 and α_1 are non-vanishing real constants of opposite sign. More complicated examples with more vector multiplets may be constructed as well. It would be interesting to study if such examples can be embedded into string theory.

A similar situation arises in the absense of hypermultiplets. As mentioned in the end of section 2.2, we can have nonvanishing moment maps that can be chosen as $P^x_\Lambda = \alpha_\Lambda \delta^{x3}$. Then we again need to satisfy the same condition $\alpha_\Lambda f_i^\Lambda = 0$ as above, and we already discussed the possible solutions.

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A Notation and conventions

We mainly follow the notation and conventions from [11]. The action is defined by $S = \int \sqrt{|g|}\mathcal{L}$. We start with the (ungauged) Lagrangian, whose Einstein-Hilbert and scalar derivative terms read

$$\mathcal{L} = -\frac{1}{2}R + g_{i\bar{\jmath}}\partial_\mu z^i \partial^\mu z^{\bar{\jmath}} + h_{\mathrm{uv}}\partial_\mu q^u \partial^\mu q^v \ . \tag{A.1}$$

We set the Newton constant $\kappa^2 = 1$. As in [11], we use a $\{+,-,-,-\}$ metric signature. To get positive kinetic terms for the scalars, we have to choose $g_{i\bar{\jmath}}$ and h_{uv} positive definite.

We compute Riemann curvature as follows[9]

$$\begin{aligned} R^\rho{}_{\sigma\mu\nu} &= \epsilon\left[\partial_\mu \Gamma^\rho_{\nu\sigma} - \partial_\nu \Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma}\right] , \\ R_{\mu\nu} &= R^\rho{}_{\mu\rho\nu} , \qquad R = g^{\mu\nu}R_{\mu\nu} , \end{aligned}$$

where $\epsilon = 1$ for Riemann spaces (the quaternionic and special Kähler target spaces) and $\epsilon = -1$ for Lorentzian spaces (space-time). The overall minus sign in the latter case is needed to give AdS spaces a negative scalar curvature. This gives a sphere in Euclidean space (with signature $\{+,+,+,+\}$) a positive scalar curvature.

The spin connection enters in the covariant derivative

$$\begin{aligned} D_\mu &= \partial_\mu - \frac{1}{4}\omega_\mu^{ab}\gamma_{ab} , \\ \omega_\mu^{ab} &= \frac{1}{2}e_{\mu c}\left(\Omega^{cab} - \Omega^{abc} - \Omega^{bca}\right) , \\ \Omega^{cab} &= \left(e^{\mu c}e^{\nu b} - e^{\mu b}e^{\nu a}\right)\partial_\mu e^c{}_\nu \ . \end{aligned}$$

The Lagrangian (A.1) is only supersymmetric if the Riemann curvature of the hypermultiplet moduli space satisfies $R(h_{\mathrm{uv}}) = -8n(n+2)$, where n is the number of hypermultiplets,

[9] Note that [12] computes quaternionic curvature with a additional factor 1/2.

so the dimension of quaternionic manifold is $4n$ (in applications to the universal hypermultiplet, we have $n = 1$ and hence $R = -24$).

Our conventions for the sigma matrices follow [11]; in particular they are symmetric and satisfy $\left(\sigma^{xAB}\right)^* = -\sigma^x{}_{AB}$, and we have the relation

$$\sigma^x_{AB}\sigma^{yBC} = -\delta_A^C\delta^{xy} + i\epsilon_{AD}\epsilon^{xyz}\sigma^{zDC} .$$

Indices are raised and lowered, on bosonic quantities, as

$$\epsilon_{AB}V^B = V_A\,, \qquad \epsilon^{AB}V_B = -V^A . \tag{A.2}$$

As mentioned in the main text, all fermions with upper $\mathrm{SU}(2)_R$ index have negative chirality and all fermions with lower index have positive chirality. We set γ_5 to be purely imaginary and then complex conjugation interchanges chirality.

B Moment maps and Killing vectors on special Kähler manifolds

In this appendix, we present some further relevant formulae that are used in the main body of the paper. First, we have defined the moment maps on the special Kähler manifold as follows. Given an isometry, with a symplectic embedding (2.9), we can define the functions

$$P_\Lambda \equiv i(k_\Lambda^i\partial_i\mathcal{K} + r_\Lambda) . \tag{B.1}$$

Since the Kähler potential satisfies (2.10), it is easy to show that P_Λ is real. From this definition, it is easy to verify that

$$k_\Lambda^i = -ig^{i\bar{\jmath}}\partial_{\bar{\jmath}}P_\Lambda . \tag{B.2}$$

Hence the P_Λ can be called moment maps, but they are *not* subject to arbitrary additive constants. Using (2.12) and (B.1), it is now easy to prove the relation

$$k_\Lambda^i g_{i\bar{\jmath}}k_\Sigma^{\bar{\jmath}} - k_\Sigma^i g_{i\bar{\jmath}}k_\Lambda^{\bar{\jmath}} = if_{\Lambda\Sigma}{}^\Pi P_\Pi\,, \tag{B.3}$$

also called the equivariance condition.

We can obtain formulas for the moment maps in terms of the holomorphic sections. For this, one needs the identities

$$k_\Lambda^i\partial_i X^\Sigma = -f_{\Lambda\Pi}{}^\Sigma X^\Pi + r_\Lambda X^\Sigma\,, \qquad k_\Lambda^i\partial_i F_\Sigma = c_{\Lambda,\Sigma\Pi}X^\Pi + f_{\Lambda\Sigma}{}^\Pi F_\Pi + r_\Lambda F_\Sigma\,, \tag{B.4}$$

which follow from the gauge transformations of the sections, see (2.9). Using the chain rule in (B.1), it is now easy to derive

$$P_\Lambda = \mathrm{e}^{\mathcal{K}}\Big[f_{\Lambda\Pi}{}^\Sigma(X^\Pi\bar{F}_\Sigma + F_\Sigma\bar{X}^\Pi) + c_{\Lambda,\Pi\Sigma}X^\Pi\bar{X}^\Sigma\Big]\,, \tag{B.5}$$

and similarly

$$k_\Lambda^i = -ig^{i\bar{\jmath}}\Big[f_{\Lambda\Pi}{}^\Sigma(\bar{f}_{\bar{\jmath}}^\Pi M_\Sigma + \bar{h}_{\Sigma\,\bar{\jmath}}L^\Pi) + c_{\Lambda,\Sigma\Pi}\bar{f}_{\bar{\jmath}}^\Pi L^\Sigma\Big]\,, \tag{B.6}$$

where we introduced $M_\Lambda \equiv \mathrm{e}^{\mathcal{K}/2} F_\Lambda$ and $h_{\Lambda\, i} \equiv \mathrm{e}^{\mathcal{K}/2}(\partial_i + \mathcal{K}_i) F_\Lambda$. The Killing vectors (B.6) are not manifestly holomorphic. This needs not be the case because otherwise we would have constructed isometries for arbitrary special Kähler manifolds, since holomorphic vector fields obtained from a (real) moment map solve the Killing equation.

In the remainder of this appendix, we prove the equivalence relation (2.26). We start from the consistency condition on the symplectic embedding of the gauge transformations, the first equation in (B.4). We eliminate r_Λ using (B.1), and rewrite it as

$$k^i_\Lambda f^\Sigma_i = -f_{\Lambda\Pi}{}^\Sigma L^\Pi - iP_\Lambda L^\Sigma \,. \tag{B.7}$$

Multiplications with L^Λ gives

$$L^\Lambda k^i_\Lambda f^\Sigma_i + iL^\Lambda P_\Lambda L^\Sigma = 0 \,. \tag{B.8}$$

It follows from contracting with $\mathrm{Im}\,\mathcal{N}_{\Sigma\Gamma}\bar{L}^\Gamma$ that

$$L^\Lambda P_\Lambda = -2i(L^\Lambda k^i_\Lambda)\, f^\Gamma_i \mathrm{Im}\,\mathcal{N}_{\Gamma\Sigma}\bar{L}^\Sigma \,,$$

or from contracting with $\mathrm{Im}\,\mathcal{N}_{\Gamma\Sigma} f^\Sigma_{\bar{\jmath}}$ that

$$L^\Lambda k^i_\Lambda = 2i(L^\Lambda P_\Lambda)\, g^{i\bar{\jmath}} L^\Gamma \mathrm{Im}\,\mathcal{N}_{\Gamma\Sigma} f^\Sigma_{\bar{\jmath}} \,.$$

Here we have used the special geometry identities on the period matrix, see e.g. [11]

$$L^\Lambda(\mathrm{Im}\,\mathcal{N})_{\Lambda\Sigma}\bar{L}^\Sigma = -\frac{1}{2}\,, \qquad f^\Lambda_i(\mathrm{Im}\,\mathcal{N})_{\Lambda\Sigma}\bar{f}^\Sigma_{\bar{\jmath}} = -\frac{1}{2}\, g_{i\bar{\jmath}} \,. \tag{B.9}$$

The equivalence

$$k^i_\Lambda L^\Lambda = 0 \quad \Leftrightarrow \quad P_\Lambda L^\Lambda = 0 \,,$$

now follows trivially.

C Commutators of supersymmetry tranformations

Equating (2.22) to zero gives an expression for the supercovariant derivative $\nabla_\mu \epsilon_A$ in terms of the matrices $T^-_{\mu\nu}$ and S_{AB}. Applying this operator twice gives

$$\begin{aligned}
\nabla_\nu \nabla_\mu \epsilon_A = & - \epsilon_{AB} D_\nu T^-_{\mu\rho}\gamma^\rho \epsilon^B && \epsilon_{AB} \\
& + T^-_{\mu\rho}\gamma^\rho T^+_{\nu\sigma}\gamma^\sigma \epsilon_A && \mathbf{1}_A{}^B \\
& + ig\epsilon_{AB} T^-_{\mu\rho}\gamma^\rho\gamma_\nu (S_{BC})^* \epsilon_C && \sigma^x{}_A{}^B \\
& - ig\epsilon^{BC} T^+_{\nu\rho}\gamma^\rho\gamma_\mu S_{AB}\epsilon_C && \sigma^x{}_A{}^B \\
& - g^2 S_{AB}(S_{BC})^* \gamma_\mu\gamma_\nu \epsilon_C \,, && \mathbf{1}_A{}^B + \sigma^x{}_A{}^B
\end{aligned}$$

where we have indicated the SU(2) structure on the right side. In (2.33), the commutator does not contain a part proportional to ϵ_{AB}. This implies $D_\rho T_{\mu\nu} = 0$. Calculation of the commutator now gives

$$[\nabla_\nu, \nabla_\mu]\epsilon_A = + T^-_{\mu\rho}\gamma^\rho T^+_{\nu\sigma}\gamma^\sigma \epsilon_A - (\mu \leftrightarrow \nu)$$

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$$
\begin{aligned}
&+ \frac{g}{2}\left(T^-_{\nu\rho}\gamma^\rho\gamma_\mu\overline{P}^x + T^+_{\nu\rho}\gamma^\rho\gamma_\mu P^x\right)\sigma^x{}_A{}^C\epsilon_C - (\mu\leftrightarrow\nu) \\
&- g^2\left(\frac{1}{4}P^x\overline{P^x}\delta^C_A - \frac{i}{4}P^x\overline{P^y}\epsilon^{xyz}\sigma^z{}_A{}^C\right)\gamma_{\mu\nu}\epsilon_C\ .
\end{aligned}
$$

We equate this to (2.33), where we use (2.24) and the condition (2.13):

$$
\begin{aligned}
[\nabla_\mu,\nabla_\nu]\epsilon_A &= -\frac{1}{4}R_{\mu\nu}{}^{ab}\gamma_{ab}\epsilon_A - \frac{ig}{2}F^\Lambda_{\mu\nu}P_\Lambda\epsilon_A + \frac{ig}{2}\sigma^x{}_A{}^B F^\Lambda_{\mu\nu}P^x_\Lambda\epsilon_B \\
&= -\frac{1}{4}R_{\mu\nu}{}^{ab}\gamma_{ab}\epsilon_A - \frac{ig}{2}F^\Lambda_{\mu\nu}P_\Lambda\epsilon_A - \frac{g}{2}\left(\overline{P^x}T^-_{\mu\nu} - P^xT^+_{\mu\nu}\right)\sigma^x{}_A{}^B\epsilon_B\ .
\end{aligned}
$$

Some algebra now yields the necessary and sufficient conditions to match the terms proportional to $\sigma^x{}_A{}^B$:

$$
\begin{aligned}
T^-_{\mu\nu}\overline{P^x} &= 0 \\
\epsilon^{xyz}P^y\overline{P^z} &= 0\,,
\end{aligned}
$$

which give the first conditions of section 2.3. The other conditions are obtained by comparing the parts proportional to $\mathbf{1}_A{}^B$.

D Metrics and field strengths

- $AdS_2 \times S^2$

  The line element, in local coordinates $\{t, x, \theta, \phi\}$, is

$$
\mathrm{d}s^2 = q_0^2\left(\mathrm{d}t^2 - \sin^2(t)\mathrm{d}x^2 - \mathrm{d}\theta^2 - \sin^2(\theta)\mathrm{d}\phi^2\right)\,,
$$

  where q_0 is a real, overall constant which determines the size of both AdS_2 and S^2. From (2.39) we find the only non-vanishing components

$$
\begin{aligned}
T^+_{tx} &= \frac{1}{2}q_0\sin(t)e^{i\alpha}\,, \\
T^+_{\theta\phi} &= -\frac{i}{2}q_0\sin(\theta)e^{i\alpha}\ .
\end{aligned}
$$

- The pp-wave

  The line element of a four-dimensional Cahen-Wallach space [33], in local coordinates $\{x^-, x^+, x^1, x^2\}$, is given by

$$
\mathrm{d}s^2 = -2\mathrm{d}x^+\mathrm{d}x^- - A_{ij}x^ix^j(\mathrm{d}x^-)^2 - (\mathrm{d}x^i)^2\,,
$$

  where A_{ij} is a symmetric matrix. Conformal flatness requires $A_{11} = A_{22}$ and $A_{12} = 0$. We denote $A_{11} = -\mu^2$ as A_{11} should be negative. This space is known as the pp-wave. From (2.39) we find the only non-vanishing components

$$
\begin{aligned}
T^+_{x^-x^1} &= \frac{\mu}{2}e^{i\alpha}\,, \\
T^+_{x^-x^2} &= -i\frac{\mu}{2}e^{i\alpha}\ .
\end{aligned}
$$

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